Midterm Exam Calculus 1

22 september 2014, 9.00-11.00.

Write on each page your name and student number, and on the first page your tutorial group. The use of annotations, books and calculators is not permitted. All answers must be supported by arguments/work. Success.

1. (a) Formulate the principle of mathematical induction.

   (b) Use the principle of mathematical induction to show that for all integers $n \geq 1$

   $$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

2. The function f is given by $f(x) = x\,e^x$. The nth derivative of f is denoted by $f^{(n)}(x)$. Prove that for all integers $n \geq 1$,

   $$f^{(n)}(x) = (x + n)\,e^x$$

3. Solve

   $$z^2 = 1 + i$$

   and sketch the solutions in the complex plane.

4. Determine all complex numbers z satisfying

   $$e^z = e^{-z}$$

5. Prove - using the precise ϵ,δ definition of a limit - that

   $$\lim_{x \to 3} (4x - 5) = 7$$

Maximum score:

1a	0.5	2:	1.8	3:	1.8	4:	1.8	5:	1.8
b	1.3								

Total: 9 +1 (free) = 10.